BIBLIOGRAPHY

1. Emets, Iu. P., The hodograph method in electrodynamics of continuous nonlinearly conducting media. PMM Vol. 31, №6, 1967.
2. Olivier, D. A. and Mitchner, M., Nonuniform electrical conduction in MHD channels. AIAA J., Vol. 5, №8, 1967.
3. Gel'fand, I. M., Some problems of the theory of quasi-linear equations. Usp. matem. nauk, Vol. 14, №2, 1959.

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SOME POSSIBLE MOTIONS OF A HEAVY SOLID IN A FLUID

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We consider the motion of a heavy solid with internal cyclical motions in a heavy ideal fluid of infinite extent under the conditions that the weight of the body and the Archimedean buoyant force form a couple, and that the impulsive force is vertical (the Chaplygin condition [1]).

Three new special cases in which the equations of motion of the above mechanical system are integrable [1 – 5] are considered. The equations in these cases admit of a system of three linear particular solutions. It is shown that all of these particular solutions are expressible in terms of elliptic functions of time, and that the rotational portion of the motions of the solid in the fluid described by these particular solutions is similar to the motion of a balanced gyrostat [6].

Algebraic solutions containing two arbitrary constants are given by Clebsch's second and third cases of integrability of the Kirchhoff-Clebsch equations [2 and 3] of the classical problem of internal motion of a solid bounded by a simply connected surface through an ideal fluid of infinite extent in all directions. These algebraic solutions immediately yield the "complete set" of four first integrals required for reduction of the problem to quadratures.

Liapunov [7] noted that Clebsch's third case of integrability could be considered as a certain limiting case of his second case. The fourth first integrals for these Clebsch cases are represented in a single form.

The fourth integrals in the classical cases of Steklov and Liapunov were reduced to a single form by Kolosov [8] and Kharlamov [9 and 10].

1. We consider the problem of motion in an unbounded ideal homogeneous imcompressible fluid of a heavy solid bounded by a simply connected surface with multiply connected cavities filled completely with an ideal fluid engaged in nonvortical motion. The Chaplygin conditions [1] apply, i. e. the weight of the body and the fluid in its cavities and the Archimedean buoyant force form a couple. We assume that the motion of the boundless fluid due to the motion of the solid in it is nonvortical and that the fluid is at rest at infinity.

To within a constant determined by the cyclical motion of the liquid filling the cavities of the solid, the kinetic energy T of such a system is given by

$$T=\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}(a_{ij}P_iP_j+b_{ij}R_iR_j+2c_{ij}P_iR_j),\qquad a_{ij}=a_{ji},\qquad b_{ij}=b_{ji}$$

where a_{ij}, b_{ij}, c_{ij} are constants specific to the given system, and where R_1, R_2, R_3 and P_1, P_2, P_3 are the projections of the impulsive force $\mathbf{R}$ and of the impulsive couple $\mathbf{P}$ of the system (without allowance for the cyclical motion of the fluid in the cavities of the solid) on the axes of the rectangular coordinate system $ox_1x_2x_3$ rigidly connected to the solid.

Denoting the projections of the translational and instantaneous angular velocities of the solid on the moving axes by U_1, U_2, U_3 and ω_1, ω_2, ω_3, we obtain the following expressions for these projections [3]:

$$U_i=\frac{\partial T}{\partial R_i},\qquad \omega_i=\frac{\partial T}{\partial P_i}\qquad (i=1,2,3)$$

Assuming that the impulsive force is directed along the line of action of the gravitational force, we can write the following equations of motion of the heavy solid in a fluid under the Chaplygin conditions [1 — 5]:

$$\frac{dR_1}{dt}+\omega_2R_3-\omega_3R_2=0\qquad (123)$$

$$\frac{dP_1}{dt}+\omega_2(P_3+g_3)-\omega_3(P_2+g_2)+U_2R_3-U_3R_2=r_2R_3-r_3R_2\qquad (123)\qquad (1.1)$$

Here $g=(g_1,g_2,g_3)$ is the vector of the kinetic moment with respect to the origin of the moving axes of the cyclical motion of the liquid in the cavities of the solid; $r=(r_1,r_2,r_3)$ is a vector proportional to the radius vector constructed from the centroid of the volume bounded by the surface of the body immersed in the unbounded fluid to the center of mass of the solid and the liquid filling its cavities.

As was shown by Zhukovskii [6], the liquid circulating in the cavities of the body can be replaced by rotating flywheels with steady relative motions, and that this can be done without altering the equations of motion of the system under consideration. Similar equations describe the inertial motion in an unbounded fluid of a solid bounded by a multiply connected surface (Kharlamov [5]).

Eqs. (1.1) have the three first integrals [2 — 5]

$$T-r_1R_1-r_2R_2-r_3R_3=\text{const},\quad R_1^2+R_2^2+R_3^2=\text{const}\qquad (1.2)$$
$$(P_1+g_1)R_1+(P_2+g_2)R_2+(P_3+g_3)R_3=\text{const}$$

2. Let us consider the special cases of integrability of Eqs. (1.1) in which the latter have a system of three linear particular solutions.

$$\sum_{j=1}^{3}(\alpha_{ij}P_j+\beta_{ij}R_j)=s_i\quad (i=1,2,3),\qquad \alpha_{ij},\ \beta_{ij},\ s_i=\text{const}\qquad (2.1)$$

With suitably chosen coordinate axes, system of solutions (2.1) can be written as [11]

$$P_i+k_iR_i=s_i\qquad (i=1,2,3)\qquad (2.2)$$

where the constants k_i and s_i must be determined.

Kharlamov [10] obtained the following conditions of existence of system of solutions (2.2) of Eqs. (1.1): (2.3)

$$b_{12}-c_{12}k_1\pm(c_{21}-a_{21}k_1)(k_2-k_1)=0,\qquad b_{12}-c_{21}k_2\pm(c_{12}-a_{12}k_2)(k_2-k_1)=0\quad (123)$$

$$b_{11}-b_{22}+(c_{22}-c_{11})k_1+(c_{22}-c_{22})k_2+a_3k_3(k_2-k_1)=0\qquad (123)\qquad (2.4)$$

$$a_{11}s_1 + a_{12}s_2 + a_{13}s_3 = \varkappa (s_1 + g_1) \qquad (123) \tag{2.5}$$

$$(c_{23} - a_{23}k_3)(s_1 + g_1) - (c_{13} - a_{13}k_3)(s_2 + g_2) = 0 \qquad (123) \tag{2.6}$$

$$r_1 + \varkappa (k_2 - k_3)(s_1 + g_1) + (c_{22} - a_{22}k_2)(s_1 + g_1) - (c_{12} - a_{12}k_2)(s_2 + g_2) = c_{11}s_1 + c_{21}s_2 + c_{31}s_3 \qquad (123) \tag{2.7}$$

$$r_1 - \varkappa (k_2 - k_3)(s_1 + g_1) + (c_{33} - a_{33}k_3)(s_1 + g_1) - (c_{13} - a_{13}k_3)(s_3 + g_3) = c_{11}s_1 + c_{21}s_2 + c_{31}s_3 \qquad (123)$$

Here $\varkappa$ is some parameter.

Relations (2.3) – (2.7) are the conditions of mechanical realizability of these motions of the solid in the fluid. If these conditions are satisfied by some real values of the constants k_i, s_i $(i = 1,2,3)$, the motions in question are possible; if they cannot be so satisfied, the motions are impossible.

From now on we shall confine ourselves to the case where

$$(k_1 - k_2)(k_2 - k_3)(k_3 - k_1) \neq 0 \tag{2.8}$$

Conditions (2.3) then yield the relations

$$c_{13} = a_{12}k_2, \quad c_{21} = a_{31}k_1, \quad b_{12} = a_{12}k_1k_2 \qquad (123) \tag{2.9}$$

This ensures satisfaction of conditions (2.6), and the expression for the double kinetic energy of the system becomes

$$2T = \sum_{(123)} [a_{11}P_1^2 + 2a_{12}P_1P_2 + 2c_{11}P_1R_1 + 2a_{12}(k_1R_1P_2 + k_2R_2P_1) + b_{11}R_1^2 + 2a_{12}k_1k_2R_1R_2] \tag{2.10}$$

Here and below the summation symbol with the index (123) means that the terms not written out are to be obtained from the given terms by cyclic permutation of their subscripts 1, 2 and 3.

Further, solving Eqs. (2.5) for s_1, s_2 and s_3, we obtain

$$s_1 = \varkappa\Delta^{-1} \{[(a_{22} - \varkappa)(a_{33} - \varkappa) - a_{23}^2]\, g_1 + [a_{23}a_{31} - a_{12}(a_{33} - \varkappa)]\, g_2 + [a_{12}a_{23} - a_{31}(a_{22} - \varkappa)]\, g_3\} \qquad (123) \tag{2.11}$$

$$\Delta = (a_{11} - \varkappa)(a_{22} - \varkappa)(a_{33} - \varkappa) - a_{23}^2(a_{11} - \varkappa) - a_{31}^2(a_{22} - \varkappa) - a_{12}^2(a_{33} - \varkappa)$$

3. Let

$$c_{11} = c_{22} = c_{33} = c \tag{3.1}$$

Eqs. (2.4) then yield the conditions
$$\frac{b_{22} - b_{33}}{a_{11}} + \frac{b_{33} - b_{11}}{a_{22}} + \frac{b_{11} - b_{22}}{a_{33}} = 0$$

$$(a_{22} - a_{33})k_2k_3 + (a_{33} - a_{11})k_3k_1 + (a_{11} - a_{22})k_1k_2 = 0$$

which give us the following equations:

$$b_{11} = b + \tau a_{22}a_{33}, \qquad k_2k_3 = \sigma + \tau a_{11} \qquad (123) \tag{3.2}$$

where b, τ and σ are arbitrary parameters. The second group of relations (3.2) yields Eqs.

$$k_1 = \varepsilon \frac{\sqrt{(\sigma + \tau a_{22})(\sigma + \tau a_{33})}}{\sqrt{\sigma + \tau a_{11}}} \qquad (\varepsilon = \pm 1) \qquad (123) \tag{3.3}$$

Subtracting and adding Eqs. (2.7) term by term and taking into account relations (2.8), (2.9), (3.1) and (3.2), we obtain $\varkappa = -\sigma / 2\tau$

$$r_1 = \frac{1}{2\tau}[2k_1k_2k_3 - \sigma(k_1 + k_2 + k_3)](s_1 + g_1) - \frac{1}{\tau}k_1k_2k_3s_1 + \frac{\sigma}{\tau}k_1s_1 - cg_1 \qquad (123) \tag{3.4}$$

Thus, if the double kinetic energy of the system is given by Formula (2.10), if Eqs. (3.1) – (3.3) hold, and if the quantities r_1, r_2 and r_3 are given by Formulas (3.4), then Eqs. (1.1) of the motion of such a solid in a fluid admit of system of linear particular solutions (2.2) in which the constants s_1, s_2 and s_3 are given by Formulas (2.11) for $\varkappa = -\sigma / 2\tau$; the problem then reduces to elliptic functions of time (see Section 6).

Specifically, the Steklov case [12] applies for $a_{12} = a_{23} = a_{31} = 0$, $g_1 = g_2 = g_3 = 0$, $r_1 = r_2 = r_3 = 0$.

4. If along with conditions (3.1) we have

$$a_{11} = a_{22} = a_{33} = a \tag{4.1}$$

then Eqs.(2.4) yield the condition

$$(b_{22} - b_{33})\, k_2 k_3 + (b_{33} - b_{11})\, k_3 k_1 + (b_{11} - b_{22})\, k_1 k_2 = 0$$

so that

$$k_2 k_3 = \tau b_{11} + \sigma / a \quad (123)$$

Substituting these equations into (2.4), we arrive at the condition $a\tau = -1$, so that

$$a k_2 k_3 = \sigma - b_{11} \quad (123) \tag{4.2}$$

From this we readily obtain

$$k_1 = \varepsilon \frac{\sqrt{(\sigma - b_{22})(\sigma - b_{33})}}{\sqrt{a(\sigma - b_{11})}} \qquad (\varepsilon = \pm 1) \qquad (123) \tag{4.3}$$

Eqs. (2.7) with allowance for (2.8), (2.9), (3.1), (4.1) and (4.2) yield Eq. $\varkappa = 1/_2 a$ and the following expressions for the quantities r_1, r_2 and r_3:

$$2r_1 = a(k_2 + k_3 - k_1)(s_1 + g_1) + (c - ak_1)\, g_1 \qquad (123) \tag{4.4}$$

Thus, if the double kinetic energy of the system is given by Formula (2.10), if Eqs. (3.1), (4.1) and (4.3) hold, and if the quantities r_1, r_2 and r_3 are given by Formulas (4.4), then Eqs. (1.1) of the motion of the solid in a fluid have system of linear particular solutions (2.2) in which the constants s_1, s_2 and s_3 are given by Formulas (2.11) for $\varkappa = 1/_2 a$; The problem then reduces to elliptic functions of time.

5. It is easy to verify the following statement. If the double kinetic energy of the system is given by (2.10), if

$$a_{11} = a_{22} = a_{33} = a, \qquad b_{11} = b_{22} = b_{33} = b, \qquad k_1 = \frac{2}{3a}(c_{22} + c_{33} - 2c_{11}) \qquad (123)$$

and if the quantities r_1, r_2, r_3 are given by the expressions

$$3r_1 = (c_{22} + c_{33} - 2c_{11})\, s_1 - (c_{11} + c_{22} + c_{33})\, g_1 \qquad (123)$$

where the constants s_1, s_2 and s_3 are given by Formulas (2.11) for $\varkappa = a / 4$, then Eqs. (1.1) of the motion of such a solid in a fluid have system of linear particular solutions (2.2), and the problem reduces to elliptic functions of time.

Specifically, for $a_{12} = a_{23} = a_{31} = 0$, $g_1 = g_2 = g_3 = 0$ and $r_1 = r_2 = r_3 = 0$ we have the integrable case established by Steklov [12].

6. Let us show that the special cases of integrability of Eqs. (1.1) cited in Sections 3 – 5 are reducible to elliptic functions of time and that the rotational portion of the motions of the heavy solid in a fluid described by these particular solutions is similar to the motion of a balanced gyrostat.

In fact, the double kinetic energy of the system is of the form (2.10) for all these particular solutions, and the constants k_1, k_2 and k_3 are related to a_{ii}, b_{ii} and c_{ii} by relations (2.4). Computing the components of the instantaneous angular velocity of the solid and making use of solutions (2.2) and relations (2.5), we obtain

$$\omega_1 = (c_{11} - a_{11}k_1) R_1 + \varkappa(s_1 + g_1) \quad (123)$$

This gives us

$$R_1 = \frac{\omega_1}{c_{11} - a_{11}k_1} - \frac{\varkappa(s_1 + g_1)}{c_{11} - a_{11}k_1} \qquad (123) \tag{6.1}$$

Substituting these expressions for R_1, R_2 and R_3 into the first group of Eqs. (1.1), we obtain the following equations for the rotation of the solid:

$$J_1 \frac{d\omega_1}{dt} = (J_2 - J_3)\,\omega_2\omega_3 + m_2\omega_3 - m_3\omega_2 \quad (123) \tag{6.2}$$

$$J_1 = \frac{1}{c_{11} - a_{11}k_1} \quad (123), \qquad m_1 = -\frac{\varkappa(s_1 + g_1)}{c_{11} - a_{11}k_1} \quad (123)$$

These equations are similar to the equations of motion of a balanced gyrostat [6]. The difference between the latter equations and those derived above consists in the fact that J_1, J_2 and J_3 are positive in the case of the balanced gyrostat, while some of our J_1, J_2 and J_3 can be negative.

Volterra [13] showed that the general solution of Eqs. (6.2) can be expressed in terms of elliptic functions of time. Hence, by virtue of relations (2.2) and (6.1), the particular solutions of Eqs. (1.1) given in Sections 3 – 5 can also be expressed in terms of elliptic functions of time.

Zhukovskii [6] provided a geometric interpretation of the motion of a balanced gyrostat. Steklov [4] showed that a similar pattern of motion is exhibited by bodies the rotational portion of whose motion is described by Eqs. (6.2) in which some of the quantities J_1, J_2 and J_3 can be negative.

7. Let us consider the case where there are no internal cyclical motions in the solid and where the solid moves in the fluid by inertia. We confine our attention to the following expression for the double kinetic energy of the system:

$$2T = \sum_{(123)} (a_1P_1^2 + b_1R_1^2 + 2c_1P_1R_1)$$

We introduce the expressions

$$J_1 = P_1 + k_1R_1 \quad (123)$$

in which the constants k_1, k_2 and k_3 are related to a_i, b_i and c_i $(i = 1,2,3)$ by relations (2.4).

It is easy to show that these expressions satisfy Eqs.

$$\frac{dJ_1}{dt} = (a_3 - a_2)\,J_2J_3 + [c_3 - c_2 + a_2(k_3 - k_1) - a_3k_3]\,R_3J_2 + [c_3 - c_2 + a_3(k_1 - k_2) + a_2k_2]\,R_2J_3 \quad (123)$$

Multiplying these equations by $k_2k_3J_1$, $k_3k_1J_2$ and $k_1k_2J_3$, respectively, and adding them term by term, we obtain Eq.

$$\frac{1}{2}\frac{d}{dt}(k_2k_3J_1^2 + k_3k_1J_2^2 + k_1k_2J_3^2) = [(a_3 - a_2)\,k_2k_3 + (a_1 - a_3)\,k_3k_1 + (a_2 - a_1)k_1k_2]\,J_1J_2J_3 + \sum_{(123)} k_3\{k_2[c_3 - c_2 + a_2(k_3 - k_1) - a_3k_3] + k_1[c_1 - c_3 + a_1(k_2 - k_3) + a_3k_3]\}\,R_3J_1J_2$$

which, on fulfillment of the conditions

$$(a_3 - a_2)\,k_2k_3 + (a_1 - a_3)\,k_3k_1 + (a_2 - a_1)\,k_1k_2 = 0 \tag{7.1}$$

$$k_3\{k_2[c_3 - c_2 + a_2(k_3 - k_1) - a_3k_3] + k_1[c_1 - c_3 + a_1(k_2 - k_3) + a_3k_3]\} = 0 \quad (123) \tag{7.2}$$

yields the integral

$$k_2k_3J_1^2 + k_3k_1J_2^2 + k_1k_2J_3^2 = \text{const} \tag{7.3}$$

Making use of relations (2.4), we can write conditions (7.2) in the form

$$(c_1 - c_2)\,(k_1 + k_2 - k_3)\,k_3 = 0 \quad (123) \tag{7.4}$$

8. Let us satisfy conditions (7.4) by setting

$$c_1 = c_2 = c_3 = c \tag{8.1}$$

Relations (2.4) and (7.1) then yield relations (3.2) and (3.3). Integral (7.3) becomes

$$\sum_{(123)} (\sigma + \tau a_1) \left[P_1 + \varepsilon \frac{\sqrt{(\sigma + \tau a_2)(\sigma + \tau a_3)}}{\sqrt{\sigma + \tau a_1}} R_1 \right]^2 = \text{const} \tag{8.2}$$

or

$$\begin{gathered}\tau (a_1 P_1^2 + a_2 P_2^2 + a_3 P_3^2 + \tau a_2 a_3 R_1^2 + \tau a_3 a_1 R_2^2 + \tau a_1 a_2 R_3^2) + \sigma^2 (R_1^2 + R_2^2 + R_3^2) + \\ + \sigma [P_1^2 + P_2^2 + P_3^2 + \tau (a_2 + a_3) R_1^2 + \tau (a_3 + a_1) R_2^2 + \tau (a_1 + a_2) R_3^2] + \\ + 2\varepsilon \sqrt{(\sigma + \tau a_1)(\sigma + \tau a_2)(\sigma + \tau a_3)} (P_1 R_1 + P_2 R_2 + P_3 R_3) = \text{const}\end{gathered} \tag{8.3}$$

Since σ is arbitrary and does not enter into Eqs. (1.1), Eq. (8.3) implies that

$$\begin{gathered}a_1 P_1^2 + a_2 P_2^2 + a_3 P_3^2 + \tau a_2 a_3 R_1^2 + \tau a_3 a_1 R_2^2 + \tau a_1 a_2 R_3^2 = \text{const} \\ P_1 R_1 + P_2 R_2 + P_3 R_3 = \text{const}, \qquad R_1^2 + R_2^2 + R_3^2 = \text{const}\end{gathered} \tag{8.4}$$

$$P_1^2 + P_2^2 + P_3^2 + \tau (a_2 + a_3) R_1^2 + \tau (a_3 + a_1) R_2^2 + \tau (a_1 + a_2) R_3^2 = \text{const} \tag{8.5}$$

Integrals (8.4) are the known Kirchhoff integrals of (1.2), and (8.5) is the fourth algebraic first integral for the second integrable case of Clebsch [3].

Thus, if the double kinetic energy of the system is given by

$$2T = \sum_{(123)} [a_1 P_1^2 + 2c P_1 R_1 + (b + \tau a_2 a_3) R_1^2]$$

and if

$$g_1 = g_2 = g_3 = 0, \qquad r_1 = r_2 = r_3 = 0$$

then Eqs. (1.1) admit of integral (8.2), which immediately yields the "complete set" of the four first integrals (8.4), (8.5) required for reduction of the problem to quadratures. The second integrable case of Clebsch [3] has been obtained.

9. In addition to conditions (8.1) let us also stipulate that

$$a_1 = a_2 = a_3 = a$$

Eqs. (2.4) then yield relations (4.2) and (4.3). Integral (7.3) can be written as

$$\sum_{(123)} (\sigma - b_1) \left[P_1 + \varepsilon \frac{\sqrt{(\sigma - b_2)(\sigma - b_3)}}{\sqrt{a (\sigma - b_1)}} R_1 \right]^2 = \text{const} \tag{9.1}$$

Since σ is arbitrary and does not appear in Eqs. (1.1), Eq. (9.1) implies that

$$\begin{gathered}a (P_1^2 + P_2^2 + P_3^2) - (b_2 + b_3) R_1^2 - (b_3 + b_1) R_2^2 - (b_1 + b_2) R_3^2 = \text{const} \\ P_1 R_1 + P_2 R_2 + P_3 R_3 = \text{const}, \qquad R_1^2 + R_2^2 + R_3^2 = \text{const}\end{gathered} \tag{9.2}$$

$$a (b_1 P_1^2 + b_2 P_2^2 + b_3 P_3^2) - b_2 b_3 R_1^2 - b_3 b_1 R_2^2 - b_1 b_2 R_3^2 = \text{const} \tag{9.3}$$

Integrals (9.2) are the Kirchhoff integrals of (1.2), and (9.3) is the fourth algebraic first integral for the third integrable case of Clebsch [3]. Thus, if

$$2T = \sum_{(123)} (a P_1^2 + 2c P_1 R_1 + b_1 R_1^2), \qquad \begin{matrix} g_1 = g_2 = g_3 = 0 \\ r_1 = r_2 = r_3 = 0 \end{matrix}$$

then Eqs. (1.1) have first integral (9.1), which immediately yields the complete set of the four first integrals (9.2), (9.3) required for reduction of the problem to quadratures. The third integrable case of Clebsch [3] has been obtained.

Thus, we have shown in Sections 7 — 9 that the fourth algebraic first integrals in the second and third integrable cases of Clebsch can be expressed in the same form (7.3).

In conclusion we note that the forms of integrals (8.2) and (9.1) "naturally" imply two special cases of integrability characterized by systems of three linear particular solutions and conforming to the assumption that the arbitrary constants in the right sides of integrals (8.2) and (9.1) are equal to zero. The first of these cases of integrability was noted by Steklov [12].

BIBLIOGRAPHY

1. Chaplygin, S. A., A new particular solution of the problem of motion of a solid in a fluid. Collected Works. Vol. 1, Leningrad, Izd. Akad. Nauk SSSR, 1933.
2 Kirchhoff, G. R., Über the Bewegung eines Körpers in einer Flüssigkeit. Z. reine und angew. Math., Vol. 71, 1870.
3. Clebsch, A., Über die Bewegung eines Körpers in einer Flüssigkeit. Math. Annalen Vol. 3, 1870.
4. Steklov, V. A., On the Motion of a Solid in a Fluid. Khar'kov, 1893.
5. Kharlamov, P. V., The motion in a fluid of a body bouded by a multiply connected surface. PMTF №4, 1963.
6. Zhukovskii, N. E., The Motion of a Solid Containing Cavities Filled with a Homogeneous Liquid. Complete Works, Vol. 3, Moscow-Leningrad, ONTI, 1936.
7. Liapunov, A. M., A New Case of Integrability of the Differential Equations of Motion of a Solid in a Fluid. Collected Works, Vol. 1, Moscow, Izd. Akad. Nauk SSSR, 1954.
8. Kolosoff, G., Sur le mouvement d'un solide dans un liquide indéfini. Compt. Rend. Sèan. l'Acad. de Scien., Vol. 169, 1919.
9. Kharlamov, P. V., A general case in which the Kirchhoff equations are integrable. Tr. Donetsk. industr. Inst., ser. mekhan. -matemat., Vol. 20, №1, 1957.
10. Kharlamov, P. V., Solutions of the equations of rigid body dynamics. PMM Vol. 29, №3, 1965.
11. Chaplygin, S. A., Some cases of motion of a solid in a fluid, 2. Collected Works, Vol. 1, Leningrad, Izd. Akad. Nauk SSSR, 1933.
12. Steklov, V. A., Some possible motions of a solid in a fluid. Tr. Otd. fiz. nauk Obshchestva liubitelei estestvoznaniia Vol. 7, 1895.
13. Volterra, V., Sur la théorie des variations des latitudes. Acta Math., Vol. 22, 1899.

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